4.5

Getting to the Root of It All

Rational Root Theorem

LEARNING GOALS

In this lesson, you will:

- Use the Rational Root Theorem to determine possible roots of a polynomial.
- Use the Rational Root Theorem to factor higher order polynomials.
- Solve higher order polynomials.

KEY TERM

- Rational Root Theorem

Out of the many vegetables there are to eat, root vegetables are unique. Root vegetables are distinguishable because the root is the actual vegetable that is edible, not the part that grows above ground. These roots would provide the plant above ground the nourishment they need to survive, just like the roots of daisies, roses, or trees; however, we pull up the roots of particular plants from the ground to provide our own bodies with nourishment and vitamins. Although root vegetables should only pertain to those edible parts below the ground, the category of root vegetables includes corms, rhizomes, tubers, and any vegetable that grows underground. Some of the most common root vegetables are carrots, potatoes, and onions.

Root vegetables were a very important food source many years ago before people had the ability to freeze and store food at particular temperatures. Root vegetables, when stored between 32 and 40 degrees Fahrenheit, will last a very long time. In fact, people had root cellars to house these vegetable types through cold harsh winters. In fact, some experts believe people have been eating turnips for over 5000 years! Now that's one popular root vegetable! So, what other root vegetables can you name? What root vegetables do you like to eat?

PROBLEM 1 Ideas Taking Root

Consider the product and sum of each set of roots.

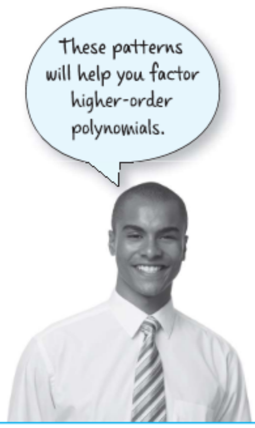
Polynomial	Roots	Product of Roots	Sum of Roots
$x^2 + 4x - 1 = 0$	$-2 \pm \sqrt{5}$	-1	-4
$x^3 + 2x^2 - 5x - 6 = 0$	$-1, 2, -3$	6	-2
$2x^3 + 5x^2 - 8x - 20 = 0$	$\pm 2, -\frac{5}{2}$	10	$-\frac{5}{2}$
$4x^3 - 3x^2 + 4x - 3 = 0$	$\pm i, \frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
$36x^3 + 24x^2 - 43x + 86 = 0$	$\frac{2}{3} \pm \frac{\sqrt{3}}{2} i, -2$	$-\frac{43}{18}$	$-\frac{2}{3}$
$4x^4 - 12x^3 + 13x^2 - 2x - 6 = 0$	$1 \pm i, -\frac{1}{2}, \frac{3}{2}$	$-\frac{3}{2}$	3

4

1. Compare the sums of the roots to the first two coefficients of each polynomial equation. What conclusion can you draw?

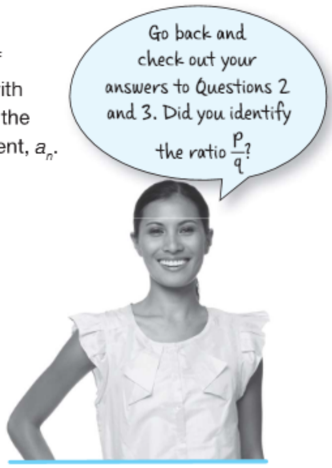
2. Compare the products of the roots to the first and last coefficients of each odd degree polynomial equation. What conclusion can you draw?

3. Compare the products of the roots to the first and last coefficients of each even degree polynomial equation. What conclusion can you draw?



Up until this point, in order to completely factor a polynomial with a degree higher than 2, you needed to know one of the factors or roots. Whether that was given to you, taken from a table, or graph and verified by the Factor Theorem, you started out with one factor or root. What if you are not given any factors or roots? Should you start randomly choosing numbers and testing them to see if they divide evenly into the polynomial? This is a situation when the *Rational Root Theorem* becomes useful.

The **Rational Root Theorem** states that a rational root of a polynomial $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0x^0$ with integer coefficients is of the form $\frac{p}{q}$ where p is a factor of the constant term, a_0, and q is a factor of the leading coefficient, a_n.



4. Beyonce and Ivy each list all possible rational roots for the polynomial they are given.

Beyonce

$4x^4 - 2x^3 + 5x^2 + x - 10 = 0$

p could equal any factors of -10 so $\pm1, \pm2, \pm5, \pm10$

q could equal any factors of 4 so $\pm1, \pm2, \pm4$

Therefore, possible zeros are

$\frac{p}{q} = \pm1, \pm2, \pm5, \pm10,$

$\pm\frac{1}{2}, \pm\frac{5}{2},$

$\pm\frac{1}{4}, \pm\frac{5}{4}.$

Ivy

$6x^3 - 2x^3 + x^2 - 3x - 15 = 0$

p could equal any factors of -15 so $\pm1, \pm3, \pm5, \pm15$

q could equal any factors of 6 so $\pm2, \pm3, \pm6$

Therefore, possible zeros are

$\frac{p}{q} = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2},$

$\pm\frac{1}{3}, \pm1, \pm\frac{5}{3}, \pm5,$

$\pm\frac{1}{6}, \pm\frac{5}{6}.$

Explain why Ivy is incorrect and correct her work.

5. Complete each step to factor and solve $x^4 + x^3 - 7x^2 - x + 6 = 0$.

a. Determine all the possible rational roots.

b. Use synthetic division to determine which of the possible roots are actual roots.

c. Rewrite the polynomial as a product of its quotient and linear factor.

4

d. Repeat steps *a*–*c* for the cubic expression.

e. Factor completely and solve.

6. Determine all roots for $x^4 - 7x^2 - 18 = 0$.

a. Determine the possible roots.

b. Use synthetic division to determine one of the roots.

4

c. Rewrite the original polynomial as a product.

d. Determine the possible rational roots of the quotient.

e. Use synthetic division to determine one of the roots.

f. Rewrite the original polynomial as a product.

g. Determine the possible rational roots of the quotient.

h. Determine the remaining roots.

i. Rewrite the original polynomial as a product.

PROBLEM 2 What Bulbs are in Your Garden?

You have learned many different ways to solve higher order polynomials. To determine all the roots or solutions of a polynomial equation:

- Determine the possible rational roots.
- Use synthetic division to determine one of the roots.
- Rewrite the original polynomial as a product.
- Determine the possible rational roots of the quotient.
- Repeat the process until all the rational roots are determined.
- Factor the remaining polynomial to determine any irrational or complex roots.
- Recall that some roots may have a multiplicity.

1. Solve each equation over the set of complex numbers.

   a. $x^3 + 1 = 0$

b. $x^4 + 3x^2 - 28 = 0$

c. $x^4 - 5x^2 - 6x - 2 = 0$

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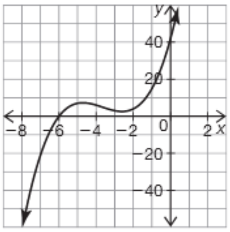
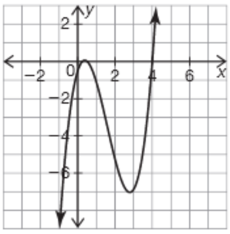
2. Determine the zeros of each function.

a. $f(x) = x^3 + 11x^2 + 37x + 42$



b. $f(x) = x^3 - 4.75x^2 + 3.125x - 0.50$



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Be prepared to share your solutions and methods.